Notation used in this booklet

X random variable that can take many values
x single observation from the random variable X
Σx The sum of all values of x

μ Population mean
$\bar{x}$ Sample mean
σ Population standard deviation
s Sample standard deviation
σ^2 Population variance
s^2 Sample variance

H_0 null hypothesis
H_1 alternative hypothesis (research hypothesis)
p probability of obtaining the observed data if H_0 is true
α significance level = probability of making a Type I error (rejecting H_0 when it is true)
β probability of making a Type II error (accepting H_0 when it is false)

Descriptive statistics

mean

$$\bar{x} = \frac{\sum x}{n}$$

population variance

$$\sigma_X^2 = \frac{\sum (x-\mu)^2}{n}$$

sample variance

$$s_X^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

population standard deviation

$$\sigma_X = \sqrt{\sigma_X^2} = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

sample standard deviation

$$s_X = \sqrt{s_X^2} = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$$

The normal distribution

Converting any normal distribution $N(\mu,\sigma^2)$ to the standard normal distribution $Z = N(0,1)$

$$z = \frac{x-\mu}{\sigma}$$

Correlation and regression

Sample covariance of two variables X and Y (the left-hand expression is the 'conceptual' formula; the right-hand one is mathematically identical but quicker to compute)

$$\mathrm{cov}_{XY} = \frac{\sum (x-\bar{x})(y-\bar{y})}{n-1} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{n-1}$$

Pearson product–moment correlation coefficient (varies from –1 to +1)

$$r_{XY} = \frac{\mathrm{cov}_{XY}}{s_X s_Y}$$

Adjusted r (always positive)

$$r_{adj} = \sqrt{1 - \frac{(1-r^2)(n-1)}{n-2}}$$

Is r significantly different from zero? A t test with n – 2 degrees of freedom

$$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

This test assumes (1) the variance of Y is roughly the same for all values of X, i.e. homogeneity of variance; (2) for all values of X, the corresponding values of Y should be normally distributed; (3) X and Y are both normally distributed. Look up the value of t in the tables of the t distribution to see if is significant.

Regression, predicting Y from X

linear regression equation

$$\hat{Y} = bX + a$$

coefficients

$$a = \bar{y} - b\bar{x} \qquad b = \frac{\mathrm{cov}_{XY}}{s_X^2} = r\frac{s_Y}{s_X}$$

Difference tests — parametric

Standard error of the mean (SEM)

$$s_{\bar{x}} = \frac{s_X}{\sqrt{n}}$$

One-sample t test and two-related-sample (paired) t test

$$t_{n-1} = \frac{\bar{x} - \mu}{s_{\bar{x}}} = \frac{\bar{x} - \mu}{\frac{s_X}{\sqrt{n}}}$$

where the null hypothesis is that $\bar{x} = \mu$. For a one-sample *t* test, $\bar{x}$ and s_X refer to the mean and standard deviation of the observations from the single sample; for a two-sample *t* test, they refer to the mean and standard deviation of the *differences* between the two samples in each pair. The *t* test has $n - 1$ degrees of freedom.

Two-sample t test for unrelated samples — where the variances of the two groups are equal

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \qquad t_{n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

where s_p^2 is the pooled variance and the null hypothesis is that $\bar{x}_1 = \bar{x}_2$. The denominator is the standard error of the differences between means (SED). The *t* test has $n_1 + n_2 - 2$ degrees of freedom. If the two samples are of equal size ($n_1 = n_2$), a simpler formula can be used:

$$t_{n_1+n_2-2} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Two-sample t test for unrelated samples — where the variances of the two groups are unequal

$$t' = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Test this just as if it were a *t* score, but with fewer degrees of freedom: **degrees of freedom = (n_1 – 1) or (n_2 – 1),** whichever is smaller.

Assumptions of t tests

(1) The *t* test assumes that the underlying populations of the scores (or difference scores, for the paired *t* test) are normally distributed.
(2) For a two-sample test, in order to use the equal-variance *t* test, we assume the two samples come from populations with equal variances ($\sigma_1^2 = \sigma_2^2$). If this is not the case, especially if $n_1 \neq n_2$, we should use the unequal-variance version of the *t* test.

The F test for differences between two variances (used to choose the form of the t test)

Put the larger variance on top of the ratio:

$$F_{n_1-1,n_2-1} = \frac{s_1^2}{s_2^2} \text{ if } s_1^2 > s_2^2 \qquad F_{n_2-1,n_1-1} = \frac{s_2^2}{s_1^2} \text{ if } s_2^2 > s_1^2$$

The subscripts on the *F* are the numbers of degrees of freedom in the numerator and denominator, respectively (either $n_1 - 1$ and $n_2 - 1$, or $n_2 - 1$ and $n_1 - 1$). If the calculated value of *F* exceeds the critical value for the relevant α and degrees of freedom, reject the null hypothesis that the two samples come from populations with equal variances, and use the *unequal variances* form of the *t* test to test for differences between the means of the two samples. If the calculated value of *F* is not significant, assume that the populations have equal variances, and use the *equal variances* form of the *t* test to test for differences between the two means.

The *F* test assumes that the underlying populations are normally distributed.

Difference tests — nonparametric

How to rank data

Place the data in ascending numerical order. Assign them ranks, starting with rank 1 for the smallest datum. If two or more data are tied for two or more ranks, assign the *mean* of those ranks to be each datum's rank.

The Mann–Whitney U test for two independent samples

1. Call the smaller group 'group 1', and the larger group 'group 2', so $n_1 < n_2$. (If $n_1 = n_2$, choose at random.)
2. Calculate the sum of the ranks of group 1 (= R_1) and group 2 (= R_2).
3. $U_1 = R_1 - \frac{n_1(n_1+1)}{2}$
4. $U_2 = R_2 - \frac{n_2(n_2+1)}{2}$
5. The Mann–Whitney statistic U is the smaller of U_1 and U_2.

As a check, verify that $U_1 + U_2 = n_1 n_2$ and $R_1 + R_2 = \frac{(n_1+n_2)(n_1+n_2+1)}{2}$. The null hypothesis is that the two samples come from identical populations.

The Wilcoxon matched-pairs signed-rank test for two related samples

1. Calculate the difference score for each pair of samples.
2. Ignore any differences that are zero.
3. Rank the difference scores, *ignoring their sign* (+ or –).
4. Add up all the ranks for difference scores that were positive; call this T^+.
5. Add up all the ranks for difference scores that were negative; call this T^-.
6. The Wilcoxon matched-pairs statistic T is the smaller of T^+ and T^-.

As a check, verify that $T^+ + T^- = \frac{n(n+1)}{2}$. The null hypothesis is that the difference scores are symmetrically distributed about zero.

The Wilcoxon signed-rank test for one sample

Calculate a difference score ($x - M$) for each score x, and proceed as above. The null hypothesis is that the scores are symmetrically distributed with a median of M.

Chi-squared (χ^2) test

Regardless of the type of test,

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$ where O = observed value, E = expected value.

For a goodness-of-fit test (one categorical variable; the expected proportions in each category are known beforehand) with c categories, there are $c - 1$ degrees of freedom.

For a contingency test (two categorical variables) with R rows and C columns, there are $(R - 1)(C - 1)$ degrees of freedom, and the expected values are given by

$$E(row_i, column_j) = \frac{R_i C_j}{n}$$

where R_i is the row total for row i, C_j is the column total for row j, and n is the total number of observations.

The χ^2 test assumes equal independence of observations, normality (no values of E less than 5), and inclusion of all observations (including non-occurrences).

Confidence intervals

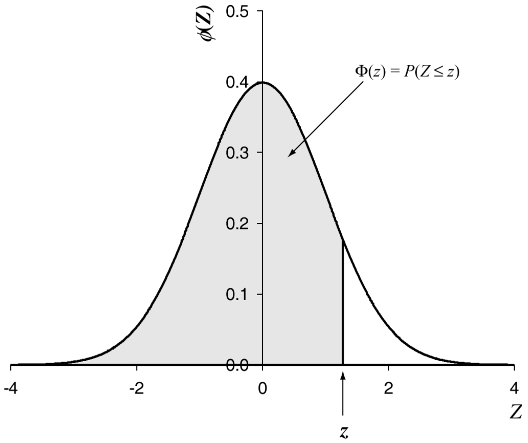
Normal distribution; population mean (μ) and SD (σ) known

Confidence intervals = $\mu \pm \sigma Z_{\text{critical}}$. (For 95% confidence intervals, $Z_{\text{critical}} = 1.96$.)

Normal distribution; sample mean and SD known

Confidence intervals = $\bar{x} \pm \frac{s_X}{\sqrt{n}} t_{critical(n-1)df}$. (For 95% confidence intervals, use t for $\alpha = 0.05$ two-tailed.)

The standard normal distribution, $Z = N(0,1)$



Mean = 0. Standard deviation = 1 (i.e. one Z point = one SD).

Cumulative distribution function $\Phi(z)$ is the area under the probability density function to the left of z (see figure).

This table gives the cumulative distribution function. **"If I know a Z score, what is the probability that a number $\leq z$ comes from a standard normal distribution?"**

If you have a Z score of 1.14, read down the left-hand side until you get to the row labelled '1.1', then read across until you get to the column labelled '0.04'. The number you reach is $\Phi(1.14)$. If you have a **negative Z score, $-z$,** calculate $1 - \Phi(z)$. For example, the probability associated with a Z score of –1.91 is (1 – 0.9719) = 0.0281. **If you want to know the probability that a number > z comes from a standard normal distribution, it's 1 minus the probability that a number $\leq z$ comes from the distribution. The 'significance level' of a Z score is the probability that a number *equal to or more extreme* than Z ($\geq z$ if z is positive, $\leq z$ if z is negative) comes from this distribution.**

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	**0.9750**	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	**0.9951**	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	**0.9995**	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Source: Microsoft Excel 97 NORMSDIST function

Probabilities corresponding to a two-tailed α of 0.05, 0.01, and 0.001 are shown in bold. (These correspond to an α for each tail of 0.025, 0.005, and 0.0005.)

Spearman's correlation coefficient for ranked data, r_s

Here are the **critical values of $|r_s|$** (the absolute magnitude of r_s, ignoring any + or – sign) for different values of n and α. If your value of $|r_s|$ is **bigger than** the critical value, you would reject the null hypothesis. (If the value shown in the table is blank, it is not possible to reject the null hypothesis, since $|r_s|$ cannot be bigger than 1.)

One-tailed α **Two-tailed α** ***n***	**0.05** **0.10**	**0.025** **0.05**	**0.01** **0.02**	**0.005** **0.01**
1				
2				
3				
4				
5	0.900			
6	0.829	0.886	0.943	
7	0.714	0.786	0.893	
8	0.643	0.738	0.833	0.881
9	0.600	0.683	0.783	0.833
10	0.564	0.648	0.745	0.794
11	0.523	0.623	0.736	0.818
12	0.497	0.591	0.703	0.780
13	0.475	0.566	0.673	0.745
14	0.457	0.545	0.646	0.716
15	0.441	0.525	0.623	0.689
16	0.425	0.507	0.601	0.666
17	0.412	0.490	0.582	0.645
18	0.399	0.476	0.564	0.625
19	0.388	0.462	0.549	0.608
20	0.377	0.450	0.534	0.591
21	0.368	0.438	0.521	0.576
22	0.359	0.428	0.508	0.562
23	0.351	0.418	0.496	0.549
24	0.343	0.409	0.485	0.537
25	0.336	0.400	0.475	0.526
26	0.329	0.392	0.465	0.515
27	0.323	0.385	0.456	0.505
28	0.317	0.377	0.448	0.496
29	0.311	0.370	0.440	0.487
30	0.305	0.364	0.432	0.478

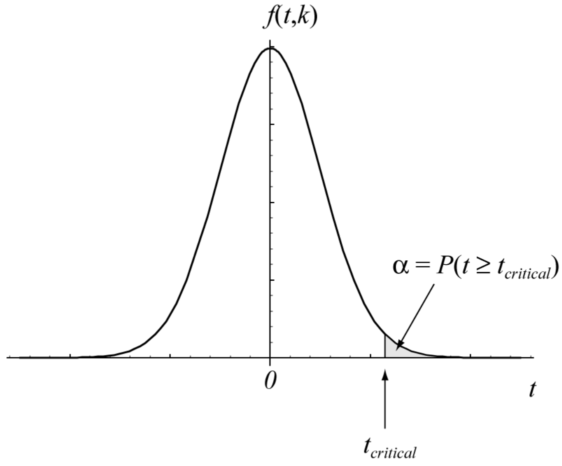
Source: Olds EG (1938), Annals of Mathematical Statistics 9. Note: there is considerable variation in published tables of critical values of $|r_s|$, because computing them is very difficult and there are many techniques for computing approximate values.

If *n* > 30, calculate a value of t instead:

$$t_{n-2} = \frac{r_s \sqrt{n-2}}{\sqrt{1-r_s^{\,2}}}$$

and test this using the tables of the t distribution with n – 2 degrees of freedom.

The *t* distribution



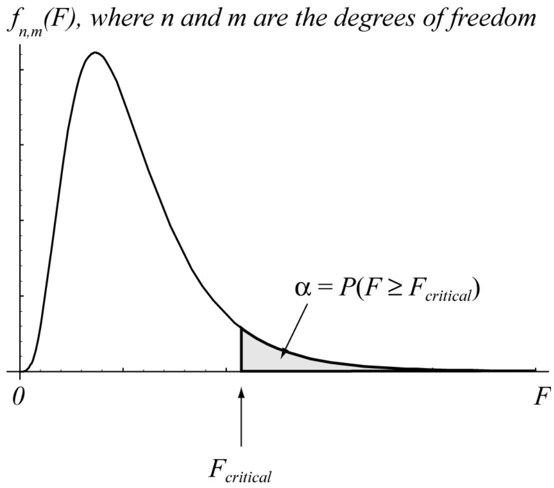
There would not be space here to give a *p* value for every possible combination of a *t* score and a certain number of degrees of freedom. So here are the **critical values** of *t* for different values of degrees of freedom and α. If your value of *t* is **bigger than** the critical value, you would reject the null hypothesis. If you have a **negative** value of *t*, just drop the minus sign (the *t* distribution is symmetrical about $t = 0$).

One-tailed α: / Two-tailed α:	0.05 / 0.1	(0.025) / 0.05	0.01 / (0.02)	(0.005) / 0.01
df				
1	6.314	12.706	31.821	63.656
2	2.920	4.303	6.965	9.925
3	2.353	3.182	4.541	5.841
4	2.132	2.776	3.747	4.604
5	2.015	2.571	3.365	4.032
6	1.943	2.447	3.143	3.707
7	1.895	2.365	2.998	3.499
8	1.860	2.306	2.896	3.355
9	1.833	2.262	2.821	3.250
10	1.812	2.228	2.764	3.169
11	1.796	2.201	2.718	3.106
12	1.782	2.179	2.681	3.055
13	1.771	2.160	2.650	3.012
14	1.761	2.145	2.624	2.977
15	1.753	2.131	2.602	2.947
16	1.746	2.120	2.583	2.921
17	1.740	2.110	2.567	2.898
18	1.734	2.101	2.552	2.878
19	1.729	2.093	2.539	2.861
20	1.725	2.086	2.528	2.845
21	1.721	2.080	2.518	2.831
22	1.717	2.074	2.508	2.819
23	1.714	2.069	2.500	2.807
24	1.711	2.064	2.492	2.797
25	1.708	2.060	2.485	2.787
26	1.706	2.056	2.479	2.779
27	1.703	2.052	2.473	2.771
28	1.701	2.048	2.467	2.763
29	1.699	2.045	2.462	2.756
30	1.697	2.042	2.457	2.750
⋮	⋮	⋮	⋮	⋮
∞	1.645	1.960	2.326	2.576

Source: Microsoft Excel 97 TINV function, except for ∞ row (NORMSDIST function)

(Explanation of '∞' entry: for ∞ *df*, critical values of *t* are the same as critical values of *z*, because the *t* distribution approaches a normal distribution as $df \to \infty$.)

The *F* distribution



There would not be space here to give a *p* value for every possible combination of a *F* score, a certain number of degrees of freedom (numerator and denominator), and α. So here are the **critical values** of *F* for different values of degrees of freedom and α. There are three tables, for $\alpha = 0.05$, $\alpha = 0.025$, and $\alpha = 0.01$. If your value of *F* is **bigger than** the critical value, you would reject the null hypothesis.

Critical values of *F*, α = 0.05 (one-tailed), equivalent to α = 0.1 if used for a two-tailed test

	Numerator *df*															
Denominator *df*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**		**15**	**20**	**25**	**30**	**40**	**50**
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	…	245.95	248.02	249.26	250.10	251.14	251.77
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	…	19.43	19.45	19.46	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	…	8.70	8.66	8.63	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	…	5.86	5.80	5.77	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	…	4.62	4.56	4.52	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	…	3.94	3.87	3.83	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	…	3.51	3.44	3.40	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	…	3.22	3.15	3.11	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	…	3.01	2.94	2.89	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	…	2.85	2.77	2.73	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	…	2.72	2.65	2.60	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	…	2.62	2.54	2.50	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	…	2.53	2.46	2.41	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	…	2.46	2.39	2.34	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	…	2.40	2.33	2.28	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	…	2.35	2.28	2.23	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	…	2.31	2.23	2.18	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	…	2.27	2.19	2.14	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	…	2.23	2.16	2.11	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	…	2.20	2.12	2.07	2.04	1.99	1.97
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	…	2.15	2.07	2.02	1.98	1.94	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	…	2.11	2.03	1.97	1.94	1.89	1.86
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	…	2.07	1.99	1.94	1.90	1.85	1.82
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	…	2.04	1.96	1.91	1.87	1.82	1.79
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	…	2.01	1.93	1.88	1.84	1.79	1.76
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	…	1.92	1.84	1.78	1.74	1.69	1.66
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	…	1.87	1.78	1.73	1.69	1.63	1.60
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	…	1.84	1.75	1.69	1.65	1.59	1.56
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	…	1.75	1.66	1.60	1.55	1.50	1.46
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	…	1.72	1.62	1.56	1.52	1.46	1.41
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	…	1.69	1.59	1.53	1.48	1.42	1.38
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	…	1.68	1.58	1.52	1.47	1.41	1.36

Source: Microsoft Excel 97 FINV function

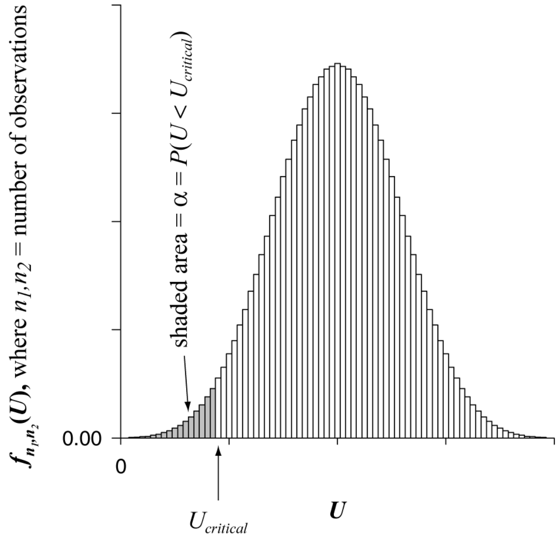
Critical values of *F*, α = 0.025, equivalent to α = 0.05 if used for a two-tailed test

	Numerator *df*																
Denominator *df*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**		**15**	**20**	**25**	**30**	**40**	**50**
1	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28	968.63	…	984.87	993.08	998.09	1001.4	1005.6	1008.1
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	…	39.43	39.45	39.46	39.46	39.47	39.48
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	…	14.25	14.17	14.12	14.08	14.04	14.01
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	…	8.66	8.56	8.50	8.46	8.41	8.38
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	…	6.43	6.33	6.27	6.23	6.18	6.14
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	…	5.27	5.17	5.11	5.07	5.01	4.98
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	…	4.57	4.47	4.40	4.36	4.31	4.28
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	…	4.10	4.00	3.94	3.89	3.84	3.81
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	…	3.77	3.67	3.60	3.56	3.51	3.47
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	…	3.52	3.42	3.35	3.31	3.26	3.22
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	…	3.33	3.23	3.16	3.12	3.06	3.03
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	…	3.18	3.07	3.01	2.96	2.91	2.87
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	…	3.05	2.95	2.88	2.84	2.78	2.74
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	…	2.95	2.84	2.78	2.73	2.67	2.64
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	…	2.86	2.76	2.69	2.64	2.59	2.55
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	…	2.79	2.68	2.61	2.57	2.51	2.47
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	…	2.72	2.62	2.55	2.50	2.44	2.41
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	…	2.67	2.56	2.49	2.44	2.38	2.35
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	…	2.62	2.51	2.44	2.39	2.33	2.30
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	…	2.57	2.46	2.40	2.35	2.29	2.25
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	…	2.50	2.39	2.32	2.27	2.21	2.17
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	…	2.44	2.33	2.26	2.21	2.15	2.11
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	…	2.39	2.28	2.21	2.16	2.09	2.05
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	…	2.34	2.23	2.16	2.11	2.05	2.01
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	…	2.31	2.20	2.12	2.07	2.01	1.97
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	…	2.18	2.07	1.99	1.94	1.88	1.83
50	5.34	3.97	3.39	3.05	2.83	2.67	2.55	2.46	2.38	2.32	…	2.11	1.99	1.92	1.87	1.80	1.75
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	…	2.06	1.94	1.87	1.82	1.74	1.70
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	…	1.94	1.82	1.75	1.69	1.61	1.56
200	5.10	3.76	3.18	2.85	2.63	2.47	2.35	2.26	2.18	2.11	…	1.90	1.78	1.70	1.64	1.56	1.51
500	5.05	3.72	3.14	2.81	2.59	2.43	2.31	2.22	2.14	2.07	…	1.86	1.74	1.65	1.60	1.52	1.46
1000	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13	2.06	…	1.85	1.72	1.64	1.58	1.50	1.45

Critical values of *F*, α = 0.01, equivalent to α = 0.02 if used for a two-tailed test

	Numerator *df*																
Denominator *df*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**		**15**	**20**	**25**	**30**	**40**	**50**
1	4052.1	4999.3	5403.5	5624.2	5763.9	5858.9	5928.3	5980.9	6022.4	6055.9	…	6156.9	6208.6	6239.8	6260.3	6286.4	6302.2
2	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39	99.40	…	99.43	99.45	99.46	99.47	99.48	99.48
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	…	26.87	26.69	26.58	26.50	26.41	26.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	…	14.20	14.02	13.91	13.84	13.75	13.69
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	…	9.72	9.55	9.45	9.38	9.29	9.24
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	…	7.56	7.40	7.30	7.23	7.14	7.09
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	…	6.31	6.16	6.06	5.99	5.91	5.86
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	…	5.52	5.36	5.26	5.20	5.12	5.07
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	…	4.96	4.81	4.71	4.65	4.57	4.52
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	…	4.56	4.41	4.31	4.25	4.17	4.12
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	…	4.25	4.10	4.01	3.94	3.86	3.81
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	…	4.01	3.86	3.76	3.70	3.62	3.57
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	…	3.82	3.66	3.57	3.51	3.43	3.38
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	…	3.66	3.51	3.41	3.35	3.27	3.22
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	…	3.52	3.37	3.28	3.21	3.13	3.08
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	…	3.41	3.26	3.16	3.10	3.02	2.97
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	…	3.31	3.16	3.07	3.00	2.92	2.87
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	…	3.23	3.08	2.98	2.92	2.84	2.78
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	…	3.15	3.00	2.91	2.84	2.76	2.71
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	…	3.09	2.94	2.84	2.78	2.69	2.64
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	…	2.98	2.83	2.73	2.67	2.58	2.53
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	…	2.89	2.74	2.64	2.58	2.49	2.44
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	…	2.81	2.66	2.57	2.50	2.42	2.36
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	…	2.75	2.60	2.51	2.44	2.35	2.30
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	…	2.70	2.55	2.45	2.39	2.30	2.25
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮	⋮	⋮	⋮	⋮
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	…	2.52	2.37	2.27	2.20	2.11	2.06
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	…	2.42	2.27	2.17	2.10	2.01	1.95
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	…	2.35	2.20	2.10	2.03	1.94	1.88
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	…	2.19	2.03	1.93	1.86	1.76	1.70
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	…	2.13	1.97	1.87	1.79	1.69	1.63
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	…	2.07	1.92	1.81	1.74	1.63	1.57
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	…	2.06	1.90	1.79	1.72	1.61	1.54

The Mann–Whitney *U* statistic



Here are the **critical values** of *U* for different values of n_1 and n_2. Only critical values for $\alpha = 0.05$ (two-tailed) are given. If your value of *U* is **smaller than** the critical value, you would reject the null hypothesis. (If the value shown in the table is zero, it is not possible to reject the null hypothesis, since *U* cannot be smaller than zero.)

Critical values of *U*, $\alpha = 0.05$ (two-tailed) or $\alpha = 0.025$ (one-tailed)

n_2 / n_1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2		0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	3	3	3	3
3			0	0	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9
4				1	2	3	4	5	5	6	7	8	9	10	11	12	12	13	14	15
5					3	4	6	7	8	9	10	12	13	14	15	16	18	19	20	21
6						6	7	9	11	12	14	15	17	18	20	22	23	25	26	28
7							9	11	13	15	17	19	21	23	25	27	29	31	33	35
8								14	16	18	20	23	25	27	30	32	35	37	39	42
9									18	21	24	27	29	32	35	38	40	43	46	49
10										24	27	30	34	37	40	43	46	49	53	56
11											31	34	38	41	45	48	52	56	59	63
12												38	42	46	50	54	58	62	66	70
13													46	51	55	60	64	68	73	77
14														56	60	65	70	75	79	84
15															65	71	76	81	86	91
16																76	82	87	93	99
17																	88	94	100	106
18																		100	107	113
19																			114	120
20																				128

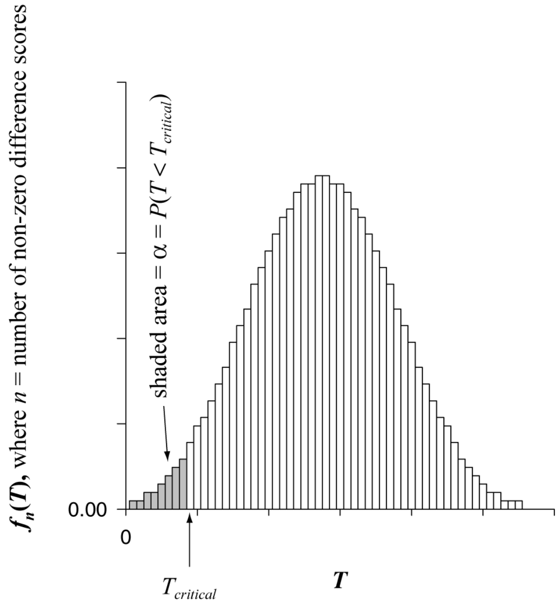
Source: R (http://www.r-project.org/), qwilcox(one-tailed α, n_1, n_2) gives q such that $P(U < q) \le \alpha$.

If $n_2 > 20$, use the normal approximation instead. Calculate a *Z* score

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

and test this using the tables of the standard normal distribution *Z*.

The Wilcoxon signed-rank *T* statistic



Here are the **critical values** of *T* for different values of *n* (where *n* is the number of non-zero difference scores) and *α*. If your value of *T* is **smaller than** the critical value, you would reject the null hypothesis. (If the value shown in the table is zero, it is not possible to reject the null hypothesis, since *T* cannot be smaller than zero.)

One-tailed *α* Two-tailed *α* *n*	0.05 0.10	0.025 0.05	0.01 0.02	0.005 0.01
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	1	0	0	0
6	3	1	0	0
7	4	3	1	0
8	6	4	2	1
9	9	6	4	2
10	11	9	6	4
11	14	11	8	6
12	18	14	10	8
13	22	18	13	10
14	26	22	16	13
15	31	26	20	16
16	36	30	24	20
17	42	35	28	24
18	48	41	33	28
19	54	47	38	33
20	61	53	44	38
21	68	59	50	43
22	76	66	56	49
23	84	74	63	55
24	92	82	70	62
25	101	90	77	69

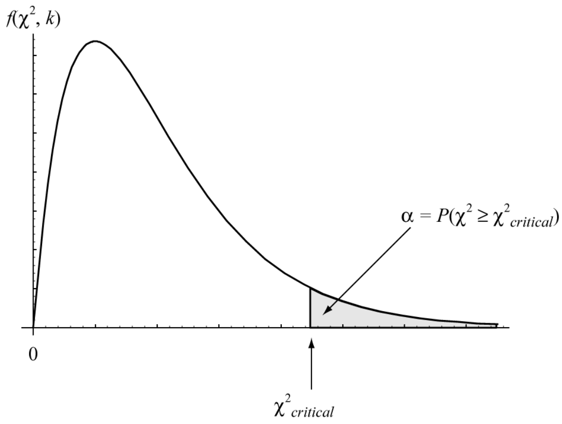
Source: R (http://www.r-project.org/), qsignrank(one-tailed α, n) gives q such that P(T < q) ≤ α.

If *n* > 25, use the normal approximation instead. Calculate a *Z* score

$$z = \frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

and test this using the tables of the standard normal distribution *Z*.

The χ^2 distribution



There would not be space here to give a *p* value for every possible combination of a χ^2 score and a certain number of degrees of freedom. So here are the **critical values** of χ^2 for different values of degrees of freedom (k) and α. If your value of χ^2 is **bigger than** the critical value, you would reject the null hypothesis.

		α	
d.f.	**0.05**	**0.01**	**0.001**
1	3.84	6.63	10.83
2	5.99	9.21	13.82
3	7.81	11.34	16.27
4	9.49	13.28	18.47
5	11.07	15.09	20.51
6	12.59	16.81	22.46
7	14.07	18.48	24.32
8	15.51	20.09	26.12
9	16.92	21.67	27.88
10	18.31	23.21	29.59
11	19.68	24.73	31.26
12	21.03	26.22	32.91
13	22.36	27.69	34.53
14	23.68	29.14	36.12
15	25.00	30.58	37.70
16	26.30	32.00	39.25
17	27.59	33.41	40.79
18	28.87	34.81	42.31
19	30.14	36.19	43.82
20	31.41	37.57	45.31
21	32.67	38.93	46.80
22	33.92	40.29	48.27
23	35.17	41.64	49.73
24	36.42	42.98	51.18
25	37.65	44.31	52.62
26	38.89	45.64	54.05
27	40.11	46.96	55.48
28	41.34	48.28	56.89
29	42.56	49.59	58.30
30	43.77	50.89	59.70
⋮	⋮	⋮	⋮
40	55.76	63.69	73.40
50	67.50	76.15	86.66
60	79.08	88.38	99.61
70	90.53	100.43	112.32
80	101.88	112.33	124.84

Source: Microsoft Excel 97 CHIINV function

(end of tables and formulae booklet)